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Forecasting Modeling and Analysis of Power Engineering in China Based on Gauss-Chebyshev Formula

Xiaojia Wang[a*]

[a]*China Key Laboratory of Process Optimization and Intelligent Decision-Making, Hefei University of Technology, Hefei, Anhui. 230009,China.*

Abstract

This paper used Gauss-Chebyshev formula to construct a new class of gray prediction model- GCGM(1,1) to overcome the lack of existed gray model and made accurate forecasting of electricity consumption for power engineering. A case study using the power engineering data of China is presented to demonstrate the effectiveness of our approach.

Keywords: Gauss Chebyshev formula; GM (1, 1) model; background value; forecasting; power engineering

1. Introduction

In power engineering, electricity consumption forecasting is known as one of the most important task in energy planning and it has great significance on management decision making for power generation groups as will as power policy adjusting for government. Therefore, it is essential to predict the electricity consumption accurately.

GM(1,1) model is the most commonly approach in electricity consumption prediction. It has the advantages of few data demand, convenient calculation and used widely consequently. However, like other prediction methods, it also has some limitations. Therefore, in recent years, research on improvement and optimization for GM(1,1) model has attracted many scholars' attention.

Literature [1] uses the conventional GM(1,1) model to forecast data. But the prediction error is relatively big. Literature [2] uses Lagrange interpolation formula to reconstruct the background value. Literature [3] uses the Newton-Cores formula to reconstruct the background value, constructs (n-1)th Newton interpolation polynomial $N(t)$ of $x^{(1)}(t)$ to calculate its value in the interval $[k, k+1]$ by the Cores formula, which is the improved background value.

Research in recent years shows that reconstruction of background value with interpolation algorithm has better performance than former trapezoid method. However, all the previous researches use certain single interpolations, such as Lagrange interpolation (ref.[2]), Newton interpolation (ref.[3]), etc. These methods can improve prediction accuracy indeed, but in order to place undue emphasis on accuracy, these methods also add nodes, which lead a vibration---Runge phenomenon that causes decrease in model applicability or even make the model cannot be used.

Based on literature mentioned above, this paper proposes a method to improve the background value by Causs-Chebyshev formula. It overcomes the problems in single interpolation methods, avoids distortion and improves the theory depth of the model. It has the advantage of high accuracy, and can solve non-equidistant nodes problem. This approach can reduce the multiply operation to (n-1) times,

* Corresponding author. Tel.:+086(0)13866720415; fax: +086(551)-2905263.
E-mail address:tonysun800@sina.com.

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which is more simple and direct to the benefit of popularizing the forecasting method in power engineering.

2. Modeling idea of conventional GM (1, 1) model

First, we introduce the modeling mechanism of traditional GM (1, 1) model.

Let $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\}$ be the original series. Make one-accumulation:

$$X^{(1)}=\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\}$$

where $X^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)\ (k=1,2,\cdots,n)$, $X^{(1)}(k)$ is the one-accumulation series of $X^{(0)}(k)$, denoted as $1-AGO$.

$x^{(1)}$ satisfies the following grey differential equation

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \tag{1}$$

where a,b are parameters. a is developing coefficient, and b is grey input.

In order to estimate a,b, discretely process (1), we have:

$$\Delta\left(x^{(1)}(k+1)+aX^{(1)}(k+1)\right)=b\quad k=1,2,\cdots,n-1 \tag{2}$$

where $\Delta\left(x^{(1)}(k+1)\right)$ is inverse accumulated generated on $(k+1)th$ and

$$\Delta\left(x^{(1)}(k+1)\right)=x^{(1)}(k+1)-x^{(1)}\left(k\right)=x^{(0)}\left(k+1\right) \tag{3}$$

In grey prediction, $x^{(1)}(k+1)$ in (2) is the background value of $dx^{(1)}/dt$ on $(k+1)th$, generally,

$$z^{(1)}(k+1)=\frac{1}{2}[x^{(1)}(k)+x^{(1)}(k+1)],(k=1,2,\cdots,n-1) \tag{4}$$

Induce (3), (4) into the following equations and get:

$$\begin{cases} z^{(1)}(2)=a\left[-\frac{1}{2}\left(x^{(1)}(1)+x^{(1)}(2)\right)\right]+b \\ z^{(1)}(3)=a\left[-\frac{1}{2}\left(x^{(1)}(2)+x^{(1)}(3)\right)\right]+b \\ \vdots \\ z^{(1)}(n)=a\left[-\frac{1}{2}\left(x^{(1)}(n-1)+x^{(1)}(n)\right)\right]+b \end{cases} \tag{5}$$

$$B=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix},Y_n=[x^{(0)}(2),x^{(0)}(3),\cdots,x^{(0)}(n)]^T\quad,\boldsymbol{\alpha}=(a,b)^T\quad,$$

Then (5) can be simplified as the linear model $Y=B\boldsymbol{\alpha}$. Using least square estimation approach, we have

$$\boldsymbol{\alpha}=\left(B^TB\right)^{-1}B^TY \tag{6}$$

Induce (6) into (1), we obtain the discrete solution:

$$\hat{x}^{(1)}(k+1)=(x^{(0)}(1)-\frac{b}{a})\cdot e^{-ak}+\frac{b}{a} \tag{7}$$

Then, we get the prediction series:

$$\hat{x}^{(0)}(k+1)=x^{(1)}(k+1)-x^{(1)}(k)=(1-e^{a})(x^{(0)}(1)-\frac{b}{a})\cdot e^{-ak} \tag{8}$$

and $k=1,2,\cdots,n$.

From (4), we know the exploit trapezoid area is

$$S(k\cdot x^{(1)}(k)\cdot x^{(1)}(k+1)\cdot(k+1))$$

When we replace the area by curve $x^{(1)}(t)$, we find the conventional GM model has some defects. As shown in Figure 1, with the index growing, data sequence changes intensify and the prediction deviation will be enlarged (ΔS), that affect the suitability of the model to some extent.

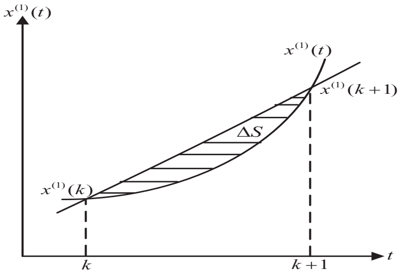

Fig. 1. Prediction deviation of GM (1, 1)

To overcome this deficiency, We use Gauss-Chebyshev formula to reconstruct the background value. Firstly, we change the form of whiten differentiation equation (1).

Do the integral operation on both sides of (1) in the interval [k,k+1] , we obtain:

$$\int_k^{k+1}\frac{dx^{(1)}}{dt}dt+a\int_k^{k+1}x^{(1)}dt=b$$

That is

$$x^{(1)}(k+1)-x^{(1)}(k)+a\int_k^{k+1}x^{(1)}d\ =tb.$$

Namely,

$$x^{(1)}(k+1)+a\int_k^{k+1}x^{(1)}dt=b \tag{9}$$

From (2) we know , the background value is

$$z^{(1)}(k+1)=\int_k^{k+1}x^{(1)}dt \tag{10}$$

3. The Gauss-Chebyshev construction of interpolation algorithm model

In order to overcoming the shortage of the above documents, we apply a new method of background value construction, the Guass-Chebyshev method, which can reduce the deviation by combing the related interpolation algorithm of numerical analysis.

Firstly, we introduce some related concepts and lemmas.

Definition 1: If every function $\varphi_k(x)\in[a,b]$ in the function system $\{\varphi_k(x)\}$ is continuous, not constant equal to zero , and satisfies the following conditions:

$$\begin{cases}(\varphi_i,\varphi_j)=\int_a^b\rho(x)\varphi_i(x)\varphi_j(x)dx=0, i\neq j\\(\varphi_i,\varphi_i)=\int_a^b\rho(x)[\varphi_i(x)]^2dx>0\end{cases} \tag{11}$$

Then , we call the function system $\{\varphi_k(x)\}$ is orthogonal function system in the interval[a,b] regarding weight function $\rho(x)$ .when $\varphi_k(x)$ is polynomial of kth degree, $\varphi(x)=\sum_{i=1}^{n}\alpha_i\varphi_i(x)$ is called orthogonal polynomial.

Definition 2: $T_n(x)=\cos(n\arccos x)$ $(-1\le x\le 1)$ is Chebyshev polynomial of kth degree, then the

weight function $\rho(x)=1/\sqrt{1-x^2}$.

Definition 3: The Interpolation quadrature formulas whose highest algebraic precision is 2n-1 are called Gauss quadrature formulas, and the quadrature nodes $x_1,x_2,\cdots,x_n$ are Guass nodes. The acquired quadrature formulas

$$\int_{-1}^{1} f(x)d \approx \sum_{k=1}^{n} A_k f(x_k) \tag{12}$$

when letting

$$\rho(x)=1, \quad [a,b]=[-1.1]$$

Definition 4: If definition 3 holds up, then Guass-Chebyshev formula equal to Guass formula, Namely

$$\int_{-1}^{1}\frac{1}{\sqrt{1-x^2}}f(x)d \approx \sum_{k=1}^{n} A_k f(x_k)$$

Lemma: Interpolating polynomial of nth degree $P_n(x)$ satisfying the condition $P(x_i)=y_i(i=0,1,\cdots,n)$ is existence and uniqueness.

Next ,we do numerical treatment to the background value $z^{(1)}(k+1)$. The algorithm steps are as follows :

Step 1: let $f(t)=x^{(1)}(t)$,then

$$\begin{aligned}\int_{k}^{k+1}x^{(1)}(t)dt &= \int_{k}^{k+1}f(t)dt=\frac{1}{2}\int_{-1}^{1}f\left(\frac{1}{2}u+k+\frac{1}{2}\right)du\\ &=\int_{-1}^{1}f(v)dv=\int_{-1}^{1}\frac{1}{\sqrt{1-v^2}}\sqrt{1-v^2}f(v)dv\\ &=\int_{-1}^{1}\frac{1}{\sqrt{1-v^2}}F(v)dv\\ &\approx A_0F(v_0)+A_1F(v_1)+A_2F(v_2)\end{aligned} \tag{13}$$

Step 2: the Guass-Node is the zero point of Chebyshev Polynomial，so that $T_3(v)=4v^3-3v=0$,and

$$v_0=-\frac{\sqrt{3}}{2}, v_1=0, v_2=\frac{\sqrt{3}}{2}$$

Step 3: $F(v)=1,v,v^2$ to the Gauss-Chebyshev polynomial with quadratic algebraic pre-cision can accurate set.

$$\begin{cases} A_0+A_1+A_2=\int_{-1}^{1}\frac{1}{\sqrt{1-v^2}}dv=\pi \\ -\frac{\sqrt{3}}{2}A_0+0A_1+\frac{\sqrt{3}}{2}A_2=\int_{-1}^{1}\frac{v}{\sqrt{1-v^2}}dv=0 \\ \frac{3}{4}A_0+0A_1+\frac{3}{4}A_2=\int_{-1}^{1}\frac{v^2}{\sqrt{1-v^2}}dv=\frac{\pi}{2} \end{cases}$$

we can obtain : $A_0=A_1=A_2=\dfrac{\pi}{3}$

Step 4: the optimal background value is the following:

$$z^{(1)}(k+1)=\int_k^{k+1}x^{(1)}(t)dt\approx\int_k^{k+1}S_k(t)dt$$
$$=\frac{\pi}{3}\sqrt{1-(-\frac{\sqrt{3}}{2})^2}x^{(1)}(k+\frac{2-\sqrt{3}}{4})+\frac{\pi}{3}\sqrt{1-(0)^2}x^{(1)}(k+\frac{1}{2})+\frac{\pi}{3}\sqrt{1-(\frac{\sqrt{3}}{2})^2}x^{(1)}(k+\frac{2+\sqrt{3}}{4}) \tag{14}$$

The formula(14) with decimal nodes operation can not implemented by computer, so we change the decimal nodes to integer nodes with suitable interpolation, the method is as follows:

$$z^{(1)}(k+1)=\int_k^{k+1}x^{(1)}(t)dt\approx\int_k^{k+1}S_k(t)dt$$
$$=\frac{\pi}{3}\sqrt{1-(-\frac{\sqrt{3}}{2})^2}x^{(1)}(k+\frac{2-\sqrt{3}}{4})+\frac{\pi}{3}\sqrt{1-(0)^2}x^{(1)}(k+\frac{1}{2})+\frac{\pi}{3}\sqrt{1-(\frac{\sqrt{3}}{2})^2}x^{(1)}(k+\frac{2+\sqrt{3}}{4}) \tag{15}$$
$$=\frac{9\pi}{32}x^{(1)}(k)+\frac{7\pi}{16}x^{(1)}(k+1)-\frac{5\pi}{96}x^{(1)}(k+2)$$

The formula (15) is new background value by improved GM (1, 1) model using Guass-Chebyshev formula.

4. Data Simulation and accuracy comparison

The main goal of this study is to predict electricity consumption of one of the province of China using Gauss-Chebyshev Formula. We first present an empirical illustration on one of the province of China's month electricity consumption forecasting to examine the performance of our grey Gauss-Chebyshev approach, and then compare the forecasting results with methods that used in reference [1], [2] and [3].

The forecasting results are as shown in Table 1.

From table 1, we can see the grey Gauss-Chebyshev prediction approach outperforms the results of reference [1], [2] and [3]. The chart of error comparison is Fig.2.

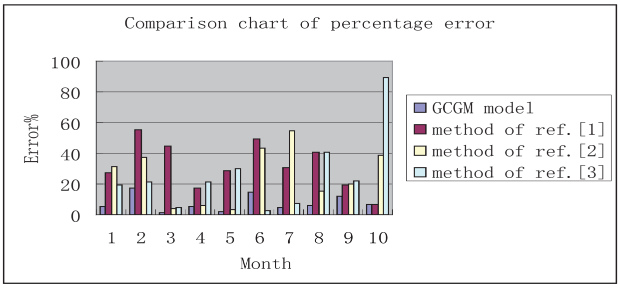

Fig.2. Comparison chart of percentage error

Table 1. Simulation results of electricity consumption forecasting

Month	Raw data	Method of this paper		Method of reference [1]		Method of reference [2]		Method of reference [3]	
		Forecasting data	RE (%)	Forecasting data	RE (%)	Forecasting data	RE (%)	Forecasting data	RE (%)
1	110852	104800	5.5	80807	27.1	76478	31.1	89635	19.1
2	135175	111371	17.6	60044	55.5	84373	37.5	91216	21.5
3	153647	94375	1.2	85342	44.4	90408	3.8	102983	4.4
4	120296	97478	5.5	141574	17.6	96873	6.1	116267	21.1
5	96362	100683	1.9	68906	28.4	103801	3.3	131265	29.7

6	90798	103993	14.5	57530	49.6	111225	43.3	148198	2.6
7	102591	107412	4.6	87676	30.8	119179	54.5	167315	7.6
8	150534	140944	6.3	89610	40.4	127702	15.1	188897	41
9	175123	154591	11.7	93649	19.1	136835	19.8	213264	21.7
10	127148	118359	6.9	110163	6.4	146621	38.9	240774	89.3
Average RE(%)		7.57		31.9		25.3		25.8	

5. Conclusion

The major contribution of this paper is to propose a new numerical analysis methodology to forecast electricity consumption of power engineering in China. The proposed Gauss-Chebyshev approach, which solved non-equidistant nodes problems, can reduce the multiply operation to (n-1) times, which is more simple and direct to the benefit of popularizing the forecasting method in power engineering. Simulation examples show a stronger applicability when using Gauss-Chebyshev formula to optimize the background value of GM (1,1) model. And will work more effectively in the power engineering and other practical applications.

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